PHASE SPACE FLOW OF PARTICLES IN SQUEEZED STATES

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Abstract

The manipulation of noise and uncertainty in squeezed states is governed by the wave nature of the quantum mechanical particles in these states. This paper uses a deterministic model of quantum mechanics in which *real* guiding waves control the flow of localized particles. This model will be used to examine the phase space flow of particles in typical squeezed states.

1 Introduction

The study of squeezed states and uncertainties is exciting. It is exciting because of its potential applications to low-noise instrumentation and communication. It is exciting because it represents a new frontier of physics, giving us new understanding of quantum mechanics. However I hear again and again that the end of many new experiments is to verify quantum mechanics. Should it really be so exciting to re-verify quantum mechanics for the ten thousandth and first time if we are so sure that the present ideas of quantum mechanics are indeed correct. Or perhaps we are still haunted by the ghosts of de Broglie and Einstein and their insistence that quantum mechanics be deterministically based. The non-verbalized and forbidden question seems to be "Is it possible to construct a localizable, deterministic model that is consistent with the observations that quantum mechanics explains so well?" And in spite of the repeated claims by physicists that they are ardent non-determinists, their pursuits seems to be strongly focused at finding a weakness in quantum mechanics or at least some inkling of a microscopic, deterministic world that has been previously hidden to physics. Certainly modern quantum optics experiments have this potential.

It is not that there is anything incorrect with quantum mechanics. It is just that it is a statistical theory and there are many situations and many applications in which it would be very useful to have a deterministic, single particle theory also. At any rate, it would seem that this workshop would be incomplete without at least one person reiterating the challenge of de Broglie and Einstein. In this vein, I will present one possible deterministic model for quantum mechanics, and then go on to relate this model to squeezed states and uncertainties. Phase diagrams will be included.

2 The Model

To begin with, this model assumes that particles have real, and localizable, continuous, existence. This is consistent with the feelings of many, if not most, physicists. However, the model also assumes, contrary to popular belief, that the waves of quantum mechanics are also *real*. After a century of discussing wave-like phenomena, perhaps the physics community might allow the possible existence of some *real* waves. I assume that a source that emits particles, also emits waves in rough proportion to the wave intensity, and anything that absorbs particles also absorbs the waves in proportion. The model also assumes that the waves entrap the ensemble (or phase space of) emitted particles and force them to flow along with the wave energy, such that a setup that splits the waves will also split the particles in rough (or statistical) proportion to the split of wave energy, thereby insuring that the particle density stays proportional to ψ^2. In a sense, the particles are nearly massless specks, carried at will by the flow of waves. All large-scale dynamics are controlled by the waves. What is there left for the particles to do? Without them, the wave fields would pass through each other. The particles represent small local, non-linear mixing sites that allow wave fields to interact with each other for the processes of scattering, transitions, absorption, emission, and detection. Thus in this model[1] propagation is determined by the waves while interactions are determined by the particles. This model is similar to models by de Broglie[2], Bohm[3], Einstein[4], and others, although it is generally more specific and physical than the previous models.

What about a wave equation? I find the easiest wave equation to work with is the Klein-Gordon Equation:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = -\hbar^2c^2\nabla^2\psi + m^2c^4\psi \tag{1}$$

which can be changed into a plasma equation:

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{\omega_o^2}{c^2}\psi = 0 \tag{2}$$

with the substitution of:

$$m = \frac{\hbar\omega_o}{c^2} \tag{3}$$

An interpretation of this is: for each species of particle, such as electrons, space supports an associative guiding wave governed by (2) with the 'plasma' frequency given by (3). There need not be any real plasma, only a space resonance at this frequency which may be fundamentally linked to the existence of the particles themselves. Furthermore, I am not hypothesizing agreement of two independent, physical quantities in (3), the mass and the plasma frequency. In quantum mechanics there is no physical, classical mass and the mass is used only as a number in various equations. In this model also there is no physical mass associated with the classical mass numbers, but instead the physics is tied up in the 'plasma frequencies'.

One can hypothesize an interaction potential between the waves and particles, and generate the phase space orbits shown in Fig. 1. In order that the orbits stay bound to the waves, even as the waves spread out and decline in amplitude, it is necessary to assume that the microscopic mass of the particles be in proportion to the waves. This can be rationalized[1] as a mass due to the energy content of a local resonance centered about the particle. Fig. 1a shows phase space orbits for particles trapped in a standing wave field. Fig. 1b shows similar orbits for traveling waves hitting a finite height barrier. Evanescent waves partially penetrate the barrier, causing some particle orbits to be carried into the barrier, while a fraction of these waves and particles tunnel completely through the barrier.

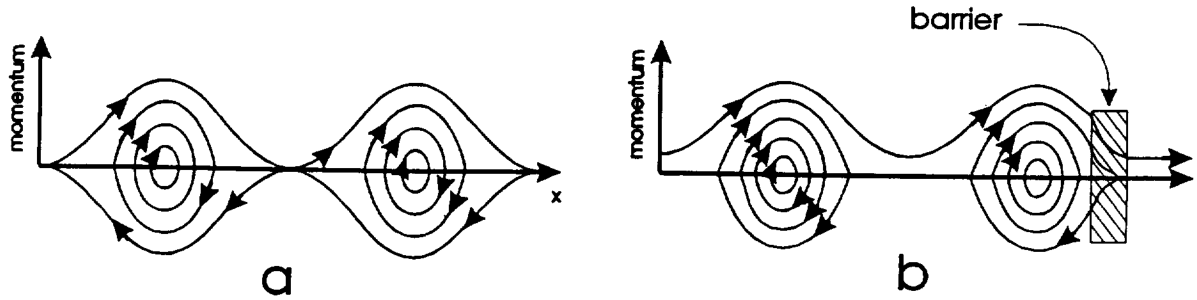

FIG. 1. Phase space orbits for particles captured in a) standing wave fields, and b) a tunneling situation. On the vertical axis is plotted the microscopic momentum of the particles which is very different from the macroscopic, observable momentum.

Just to mention a few more details of this model from Ref.1: mixing equations show that during interactions, frequency sums and vectorial wave number sums are conserved and form the basis for macroscopic energy and momentum conservation. Microscopic energy and momentum, are not so useful since the waves and the associated microscopic energy tend to spread out while propagating. The spread out remnant wave energy creates the vacuum fields, as well as being redeposited around particles, i.e. in populated states.

3 Uncertainties

The Heisenberg uncertainty principle states that:

$$\Delta x\, \Delta p \geq \hbar\ . \tag{4}$$

However, since in this model, waves control particle motion, in order to localize a particle or group of particles, we need to first localize the entrapping *wave*. The correct uncertainty relation for localizing waves is given by:

$$\Delta x\, \Delta k \geq 2\pi\ , \tag{5}$$

where δk is the spread in wave number. Just as quantum mechanics operationally does, this model replaces macroscopic momentum with wave number. Similarly, the related uncertainty:

$$\Delta t\, \Delta E \geq \hbar\ , \tag{6}$$

can be visualized as in Fig. 2, where a traveling wave pulse is entrapping particle orbits to itself. Suppose there is a detector at some point to the right that the pulse will impinge on and that we are discussing the δt in the time spread of the detected particles. In order to narrow the time spread of particles, we must narrow the time duration the *wave* pulse will be at the detector and this requires use of the relationship:

$$\Delta t\, \Delta \omega \geq 2\pi\ . \tag{7}$$

This is again like conventional quantum mechanics (at least operationally): in this case replacing energy with a frequency. So we see that the Heisenberg uncertainty relations fall out very naturally from this model.

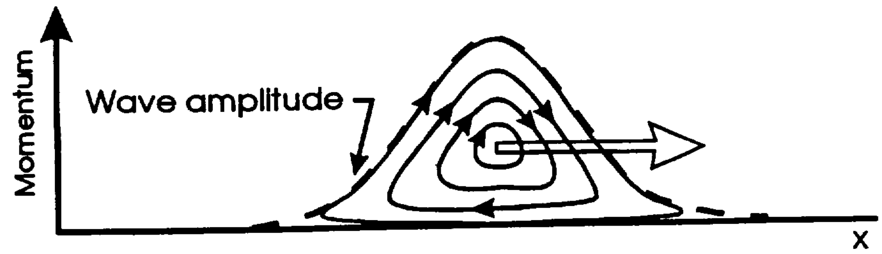

FIG. 2. Sketch of phase space of particles trapped in a traveling wave pulse.

4 Squeezed States: Amplitude and Phase

One of the most common squeezing operations involves reducing the amplitude fluctuations in a beam of particles. Fig. 3 shows a hypothetical micro-phase space of a beam. Here we assume a perfectly level traveling wave field (of completely well defined amplitude and phase) and illustrate the statistical distribution of particles that might be filling this wave field. The particles are randomly placed with respect to position and microscopic phase, and move horizontally to the right (on unplotted) orbits on the plot. The irregular line plots the fluctuating density of these particles as a function of position, which 'squeezing' is often called on to smooth out. If we selectively amplify or attenuate certain parts of the beam to level the density of particles, two undesirable things will also occur. First, we will not (in most cases we cannot) also homogenize the beam in the microscopic momentum coordinate. This will adversely effect certain other measurements, such as phase as we shall shortly see. Secondly, any attenuation or amplification process that changes particle density necessarily also attenuates or amplifies the associated wave. If the wave started out perfect, it will be degraded by this process.

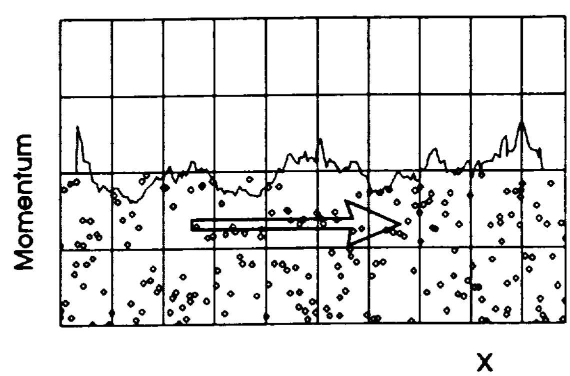

FIG. 3. Hypothetical micro-phase space of a beam of particle entrapped by a uniform traveling wave field.

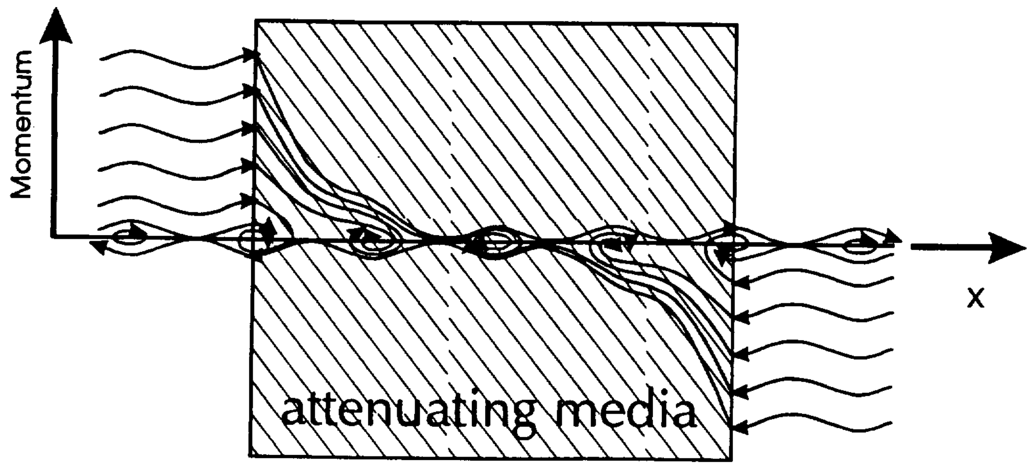

FIG. 4. Elemental phase detector showing mapping of various regions of phase space of the incoming beams into different parts of the interference pattern.

Figure 4 shows an elemental or simplistic phase detector. It involves injecting two signals into opposite ends of an attenuating medium and looking for spacial interference in the middle. A moving detector would be used to map out the particle density as a function of position to determine the relative phase of the two beams. Changing the relative phase of the two signals will move the interference maxima. This scheme was chosen because, unlike most other schemes, all particle paths are along a single line, i.e. in one dimension. This frees the other dimension for plotting microscopic momentum and greatly simplifies our phase space plotting. The interesting feature of the plot is that the phase space orbits map different microscopic momentum regions of the incoming signals into different interference maxima. In the figure here, only the uppermost momentum particles make it into the central, most useful maximum. This illustrates that inhomogeneities in the particle density in the momentum direction will adversely effect the interference pattern and thus the phase. Irregularities in the waves themselves will also have a similar adverse effect.

5 Postscript: Modern Non-local Experiments

Dr. Shimony raised the question about recent non-locality experiments being in conflict with the model presented here. My answer was that the experiments I had examined carefully[5] are in fact not inconsistent with this model. I wish to expand on that here. For example, the Franson experiment can be explained deterministically[6][7]. Even with delayed choice experiments where beams are changed after they become separated from each other, a change in the beam will affect both the particles in that beam and the waves around the particles. Since the waves determine the dynamics, it is not surprising that we get wave-like behavior in going through subsequent filters, beam splitters, and polarizers.

Surprisingly, the hardest experiments to reconcile with this model are photon cancellation experiments[8]. However even for this there is a possible classical explanation [5]: the IF filters, which most quantum optics experiments use, are resonators and as such selectively accept and reject photons. The math of the time correlation of this classical acceptance and rejection process has an uncanny resemblance to the math of entangled states. In a nutshell, when a Mach-Zehnder interferometer is balanced so as to prevent coincident detection through its outputs, the wave fields (of this model, as well as those of normal quantum mechanics) are phased so as to load up only one IF filter at a time, thereby preventing the passage photons through the IF filters of both detectors simultaneously. The conclusion is that these experiments are not conclusive proof of non-locality, particularly if one has a local model, such as the one presented here, where the particles are entrapped by and guided by waves.

These effects of IF filters also should be considered with reference to the Franson and the delayed choice type experiments discussed above to give mathematically correct agreement between this model and experiments. We shall all look forward to the day when the quantum optics experiments are done with no IF filters after the down conversion process. Also, detection of practically all photons entering the experimental apparatus is essential if one wishes to analyze the system one-dimensionally (with branches). This requires large (no lost beam due to collimation or finite detectors, etc), high-efficiency detectors with large angular acceptance. Like good accountants, we need to see where everything is going.

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